General Certificate of Education
January 2006
Advanced Level Examination

MATHEMATICS **MFP4**
Unit Further Pure 4

Monday 23 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MFP4.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Describe the geometrical transformation defined by the matrix

$$\begin{bmatrix} 0.6 & 0 & 0.8 \\ 0 & 1 & 0 \\ -0.8 & 0 & 0.6 \end{bmatrix}$$

(3 marks)

2 The matrices **P** and **Q** are defined in terms of the constant k by

$$\mathbf{P} = \begin{bmatrix} 3 & 2 & 1 \\ 1 & -1 & k \\ 5 & 3 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 5 & 4 & 1 \\ 3 & k & -1 \\ 7 & 3 & 2 \end{bmatrix}$$

(a) Express $\det \mathbf{P}$ and $\det \mathbf{Q}$ in terms of k. *(3 marks)*

(b) Given that $\det(\mathbf{PQ}) = 16$, find the two possible values of k. *(4 marks)*

3 (a) The plane Π has equation $\mathbf{r} = \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$.

(i) Find a vector which is perpendicular to both $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 4 \\ -1 \\ 0 \end{bmatrix}$. *(2 marks)*

(ii) Hence find an equation for Π in the form $\mathbf{r} \cdot \mathbf{n} = d$. *(2 marks)*

(b) The line L has equation $\left(\mathbf{r} - \begin{bmatrix} -1 \\ 2 \\ 6 \end{bmatrix} \right) \times \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix} = \mathbf{0}$.

Verify that $\mathbf{r} = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix} + t \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$ is also an equation for L. *(2 marks)*

(c) Determine the position vector of the point of intersection of Π and L. *(4 marks)*

4 The vectors **a**, **b** and **c** are given by

$$\mathbf{a} = \mathbf{i} - \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 3\mathbf{j} - \mathbf{k} \quad \text{and} \quad \mathbf{c} = 4\mathbf{i} - \mathbf{j} + 5\mathbf{k}$$

(a) (i) Evaluate $\begin{vmatrix} 1 & -1 & -1 \\ 2 & 3 & -1 \\ 4 & -1 & 5 \end{vmatrix}$. *(2 marks)*

(ii) Hence determine whether **a**, **b** and **c** are linearly dependent or independent. *(1 mark)*

(b) (i) Evaluate **b . c**. *(2 marks)*

(ii) Show that $\mathbf{b} \times \mathbf{c}$ can be expressed in the form $m\mathbf{a}$, where m is a scalar. *(2 marks)*

(iii) Use these results to describe the geometrical relationship between **a**, **b** and **c**. *(1 mark)*

(c) The points A, B and C have position vectors **a**, **b** and **c** respectively relative to an origin O. The points O, A, B and C are four of the eight vertices of a cuboid. Determine the volume of this cuboid. *(2 marks)*

5 The transformation T maps (x, y) to (x', y'), where

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

(a) Describe the difference between *an invariant line* and *a line of invariant points* of T. *(1 mark)*

(b) Evaluate the determinant of the matrix $\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ and describe the geometrical significance of the result in relation to T. *(2 marks)*

(c) Show that T has a line of invariant points, and find a cartesian equation for this line. *(2 marks)*

(d) (i) Find the image of the point $(x, -x + c)$ under T. *(2 marks)*

(ii) Hence show that all lines of the form $y = -x + c$, where c is an arbitrary constant, are invariant lines of T. *(2 marks)*

(e) Describe the transformation T geometrically. *(3 marks)*

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6 (a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a)$$ *(5 marks)*

(b) (i) Hence, or otherwise, show that the system of equations

$$\begin{aligned} x + y + z &= p \\ 3x + 3y + 5z &= q \\ 15x + 15y + 9z &= r \end{aligned}$$

has no unique solution whatever the values of p, q and r. *(2 marks)*

(ii) Verify that this system is consistent when $24p - 3q - r = 0$. *(2 marks)*

(iii) Find the solution of the system in the case where $p = 1$, $q = 8$ and $r = 0$. *(5 marks)*

7 The matrix $\mathbf{M} = \begin{bmatrix} 1 & -1 & 1 \\ 3 & -3 & 1 \\ 3 & -5 & 3 \end{bmatrix}$.

(a) Given that $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ are eigenvectors of $\mathbf{M}$, find the eigenvalues corresponding to $\mathbf{u}$ and $\mathbf{v}$. *(5 marks)*

(b) Given also that the third eigenvalue of $\mathbf{M}$ is 1, find a corresponding eigenvector. *(6 marks)*

(c) (i) Express the vector $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in terms of $\mathbf{u}$ and $\mathbf{v}$. *(1 mark)*

(ii) Deduce that $\mathbf{M}^n \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \lambda^n \mathbf{u} + \mu^n \mathbf{v}$, where λ and μ are scalar constants whose values should be stated. *(4 marks)*

(iii) Hence prove that, for all positive **odd** integers n,

$$\mathbf{M}^n \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2^n \\ 0 \\ 2^n \end{bmatrix}$$ *(3 marks)*

END OF QUESTIONS

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